Shedding Light on the Symmetries of Dark Matter

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I consider symmetries which could explain observed properties of dark matter, namely, its stability on Gyr time scales or its relic density and discuss how such symmetries can be discovered through the study of the propagation and polarization of light in its transit through dark matter.

1. Preamble

The existence of dark matter is established from galactic to cosmological distance scales if gravity is understood. The nature of dark matter is unknown, but we do know most of it must be stable or effectively so on Gyr time scales, not "hot" if it is a thermal relic, i.e., not relativistic at the time it decoupled from ordinary matter in the cooling early Universe, and sufficiently weakly interacting that it possesses no substantial strong or electromagnetic charge. As yet unknown symmetries in the dark sector could explain these features. In this contribution, based, in part, on work performed in collaboration with David C. Latimer [1], I discuss direct detection schemes [1,2,3] which can establish their existence.

The Standard Model provides no suitable dark-matter candidate, but theories which resolve the hierarchy problem to make the weak scale technically natural can. In the Minimally Supersymmetric Standard Model (MSSM), e.g., the dark-matter candidate is a Weakly Interacting Massive Particle (WIMP). Although the stability requirement is added to the MSSM through the imposition of a discrete symmetry, a WIMP with a mass of $\mathcal{O}(100\,\mathrm{GeV})$ is compatible with the observed dark-matter density. However, it is also possible to reproduce it with lighter particles which possess stronger, i.e., weak but not of G_F strength, mutual interactions [4].

If dark matter is not made of WIMPs, its stability need not be guaranteed by a discrete symmetry, and its relic density need not be fixed by thermal freezeout. What mechanisms then are operative and how do we discover them? Its stability may be guaranteed by a hidden gauge symmetry. E.g., dark matter can possess a hidden U(1) symmetry. If the gauge mediator is massless, although this is not a necessary condition, dark matter can have a *millicharge* [5,6]. If we determine that dark matter has a millicharge, we establish that dark matter is stable by dint of a gauge symmetry, much as the electron is stable — it

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cannot decay and conserve its electric charge. We can discover a dark-matter millicharge from the appearance of dispersive effects in the speed of light, and we shall discuss how the light curves of Gamma-Ray Bursts (GRBs) can be used to this purpose [1]. Moreover, its relic density may not be a numerical accident; it could be related to the fraction of the cosmological energy density in baryons Ω_B [7,8]. If so, dark matter ought be asymmetric in its particle-antiparticle content. The dark matter of such models is built of Dirac particles and can thus possess a magnetic moment. We can discover this through use of the gyromagnetic Faraday effect [2,3] and can indeed establish asymmetric dark matter, as we shall discuss.

2. Dispersive Effects in Light Propagation

To discover the nature of dark matter we must probe its couplings to known matter. E.g., in many models, dark matter can annihilate to photons and leptons, and recently much attention has been paid to the possibility of indirect dark-matter detection via "anomalous" lepton excesses in high-energy cosmic ray data. Models which yield annihilation of a dark-matter particle χ to photons through $\chi\chi \to \gamma\gamma$ also produce, by crossing, a non-zero forward Compton amplitude and thus predicate an index of refraction $n(\omega)$ which can deviate from unity. This, in turn, can yield energy-dependent, or dispersive, effects in light propagation. We can thus study the light curves of GRBs with cosmological distance to hunt for dark matter directly — we refer to Ref. [1] for all details.

We analyze the GRB data in a model-independent way by employing an effective theory analysis. To realize this, we suppose that the photon energy ω is small compared to $\omega_{\rm th}$, the threshold energy required to materialize the particles to which the dark matter can couple. If dark matter is connected to weak-scale physics, then crudely $\omega_{\rm th} \sim \mathcal{O}(100\,{\rm GeV})$. We can then expand the forward Compton amplitude in a power series in ω for $\omega \ll \omega_{\rm th}$; the symmetries of the forward Compton amplitude allow us to codify the terms which appear. Under Lorentz symmetry and P, T, and C invariance, the forward Compton amplitude for $\gamma(k) + \chi(p)$ scattering in the dark-matter rest frame is [9,10,11]

$$\mathcal{M}_r(k,p \to k,p) = f_1(\omega)\epsilon'^{\,*} \cdot \epsilon + i f_2(\omega)\mathcal{S} \cdot \epsilon'^{\,*} \times \epsilon\,, \tag{1}$$

where ϵ (ϵ') is the photon polarization in the initial (final) state and $\mathcal{S}$ is the dark-matter spin operator. The amplitude $\mathcal{M}_r(k,p \to k,p)$ is implicitly a 2×2 matrix in the photon polarization. Only its diagonal matrix elements describe dispersive effects in propagation, and thus only f_1 matters. Under analyticity and unitarity, we have a dispersion relation for f_1 [9,10], where

$${\rm Re} f_1(\omega) - {\rm Re} f_1(0) = \frac{4M\omega^2}{\pi}\int_0^\infty {\rm d}\omega' \frac{\sigma(\omega')}{\omega'^2 - \omega^2}\,, \tag{2}$$

and the optical theorem has been used to replace ${\rm Im} f_1(\omega)$ with the unpolarized cross section σ. The integral implicitly begins at $\omega_{\rm th}$, so that for $\omega \ll \omega_{\rm th}$ we have finally

$${\rm Re}\, n(\omega) = 1 + \frac{\rho}{4M^2\omega^2}\left(A_0 + A_2\omega^2 + \dots\right)\,, \tag{3}$$

where $A_0 = {\rm Re} f_1(0)$ and $A_i > 0$. Moreover, ρ is the mass density and M is the particle mass of the dark matter. A low-energy theorem fixes $A_0 = -2\varepsilon^2 e^2$ for dark matter of

electric charge εe [12]. Light emitted from a source at a distance l from us possesses a frequency-dependent arrival time $t(\omega)$ after transit through dark matter: $t(\omega) = l(\tilde{n} + \omega \mathrm{d}\tilde{n}/\mathrm{d}\omega)$, where $\omega = k/\tilde{n}$ and $\tilde{n} \equiv \mathrm{Re}\, n$. We must take the cosmological expansion into account as well [13], so that at red shift z the photon energy is blue shifted by a factor of $1+z$ relative to its present-day value ω_0. Thus

$$t(\omega_0, z) = \int_0^z \frac{\mathrm{d}z'}{H(z')}\left(1 + \frac{\rho_0(1+z')^3}{4M^2}\left(\frac{-A_0}{\left((1+z')\omega_0\right)^2} + A_2 + 3A_4(1+z')^2\omega_0^2 + ...\right)\right) \tag{4}$$

with the Hubble rate $H(z') = H_0\sqrt{(1+z')^3\Omega_M + \Omega_\Lambda}$, so that WMAP parameters characterize both the matter density and light travel time. We use the combined analysis of the WMAP five-year data and more as per Ref. [14] in the ΛCDM cosmological model, namely, $H_0 = 70.5 \pm 1.3\,\mathrm{km\ s^{-1}Mpc^{-1}}$, whereas the fraction of the energy density in matter relative to the critical density today is $\Omega_M = 0.274 \pm 0.015$ and the fraction of the energy density in the cosmological constant Λ is $\Omega_\Lambda = 1 - \Omega_M$. Various strategies must be employed to isolate the A_i; here we focus on A_0, which is fixed by the dark-matter electric charge. Although the non-observation of frequency-dependent time lags in-vacuo from GRB data have been suggested as a means to limit the appearance of Lorentz violation [15], the red-shift and frequency dependence of the dark-matter and Lorentz violation scenarios are very different. We employ, however, the statistical analysis suggested in the latter context to separate propagation and GRB source effects [16].

Gamma-Ray Bursts (GRBs) are very bright objects which are still appreciable at cosmological distances. Fermi expects to discover 200 per year [17]. GRBs possess several properties which correlate with their luminosity, so that they can be used to probe the Hubble diagram at large z and to study the properties of dark energy. A study of 69 GRBs extends the Hubble diagram to $z > 6$ and is consistent with the usual concordance model [18], which supports the use of the GRB data set in our current context. Fits to the various A_i require observations of different energies; in particular, to constrain A_0 we require observations in the radio. To select the GRBs to be included in our fit we demand that the energy of the GRB be compatible with the energy range of the Fermi Gamma-ray Burst Monitor (GBM) and that the radio flux detection be in the right location, be new, and be significantly non-zero. Moreover, we pick GRBs for which z is measured. We find 53 GRBs in all to consider and include detected radio frequencies of 75 GHz or less in our analysis. Our observable is $\tau = t(\omega_0^{\mathrm{low}}, z) - t(\omega_0^{\mathrm{high}}, z)$, where $\omega \equiv \omega_0^{\mathrm{low}}$ henceforth. Thus we fit

$$\frac{\tau}{1+z} = \tilde{A}_0 \frac{K(z)}{\nu^2} + \delta((1+z)\nu)\,. \tag{5}$$

Note the frequency $\nu \equiv \omega/2\pi$ and $K(z) \equiv (1+z)^{-1}\int_0^z \mathrm{d}z\,(1+z')H(z')^{-1}$, whereas $4\pi^2\tilde{A}_0 = -A_0\rho_0/4M^2 = 2\pi\alpha\varepsilon^2\rho_0/M^2$ and $\rho_0 \simeq 1.19 \times 10^{-6}\ \mathrm{GeV/cm^3}$ [14]. The function $\delta((1+z)\nu)$ allows for a frequency-dependent time lag for emission from the GRB in the GRB rest frame. To provide a context, we first consider the value of $|\varepsilon|/M$ which would result were we to attribute the time lag associated with the radio afterglow of one GRB to a propagation effect. Choosing the GRB with the largest value of $K(z)/\nu^2$, we have a time lag of $2.700 \pm 0.006\,\mathrm{day}$ associated with GRB 980703A at $z = 0.967 \pm 0.001$ measured at a frequency of $\nu = 1.43\,\mathrm{GHz}$. With Eq. (5), setting $\delta = 0$, and noting that

$K(z)/\nu^2 = 1170 \pm 10\,\mathrm{Mpc\,GHz}^{-2}$ if the errors in its inputs are uncorrelated, the measured time lag fixes $|\varepsilon|/M \simeq 9 \times 10^{-6}\,\mathrm{eV}^{-1}$. Since there are no known examples of a radio afterglow preceding a GRB, this one observation in itself represents a conservative limit. Turning to our fit, we include all observations in our GRB sample with frequencies of $4.0 - 75$ GHz in the GRB rest frame. A scale factor in the uncertainty in $\tau/(1+z)$ of 450 yields $\chi^2/\mathrm{ndf} = 1.13$, with $\tilde{A}_0 = 0.0010 \pm 0.0019$ day $\mathrm{GHz}^2\,\mathrm{Mpc}^{-1}$ and $\delta = 0.65 \pm 0.10$ day. Thus $\tilde{A}_0 < 0.005$ day $\mathrm{GHz}^2\,\mathrm{Mpc}^{-1}$ at 95% CL, and we determine

$$|\varepsilon|/M < 1 \times 10^{-5}\ \mathrm{eV}^{-1} \quad \mathrm{at}\, 95\%\mathrm{CL}\,, \tag{6}$$

which is comparable to the limit derived from a single observation of GRB 980703A. Our fit uses radio observations at no less than 4 GHz in the GRB rest frame, so that the associated limit is operative if $\omega_{\mathrm{th}}/2\pi > 4\,\mathrm{GHz}$, or, crudely, if $M > 8 \times 10^{-6}\,\mathrm{eV}$. We find a very large scale factor; this may stem, in part, from the circumburst environment [19].

We have found a direct observational limit on the electric-charge-to-mass ratio of dark matter. Millicharged matter limits also follow from the nonobservation of the effects of millicharged particle production. The strongest such bound from laboratory experiments is $|\varepsilon| < 3 - 4 \times 10^{-7}$ for $M \leq 0.05$ eV [20], so that for $M \sim 0.05\,\mathrm{eV}$ the limits are crudely comparable. Indirect limits also emerge from stellar evolution constraints, for which the strongest is $|\varepsilon| < 2 \times 10^{-14}$ for $M < 5\,\mathrm{keV}$ [21], as well as from the manner in which numerical simulations of galactic structure confront observations [22,23]. Such limits can be evaded; in some models, the dynamics which gives rise to millicharged matter are not operative at stellar temperatures [24]; other models evade the galactic structure constraints [25]. We estimate that our limit would have to improve by $\mathcal{O}(2 \times 10^{-3})$ before the contribution from ordinary charged matter, namely, from free electrons, could be apparent. One can expect linear improvement in the limit on ε/M as ν decreases; the observation of prompt radio emission predicted to exist at 30 MHz from GRBs, planned by the GASE collaboration [26], could yield considerably stronger limits.

3. Gyromagnetic Faraday Rotation

Light in a medium with free magnetic moments can become *circularly birefringent* if the applied magnetic field $|\mathbf{B_0}|$ is non-zero. If $|\mathbf{B_0}|$ induces a magnetization, $\mathcal{M}_0$, and the light is directed along the magnetic field, initially linearly polarized light can exhibit Faraday rotation after transit through the medium [27]. In this case, the magnetic field associated with the propagating light wave induces a component of the magnetization perpendicular to $\mathcal{M}_0$; consequently, the wave vector in the medium depends on the helicity of the light. Referring to Refs. [2,3] for all details, we note, namely, that

$$k_\pm = \omega \sqrt{1 \pm \frac{\chi_0 \omega_B}{\omega \pm \omega_B}}\,, \tag{7}$$

where $\chi_0 \equiv \mathcal{M}_0/B_0$, $\omega_B \equiv g\mu_M B_0$, and $\hbar = c = 1$. The magnetic moment μ of a particle of mass M is $\mu = Sg\mu_M$ with $\mu_M \equiv e/2M$, where S is its spin and g is its Landé factor. Expanding in ω_B/ω, we find

$$k_{\mathrm{diff}} \equiv k_+ - k_- = \chi_0 \omega_B + \frac{\chi_0 \omega_B^3}{\omega^2} + \frac{\chi_0^2 \omega_B^3}{2\omega^2} + \dots\,, \tag{8}$$

which engenders Faraday rotation, and

$$k_{\rm avg} \equiv \frac{1}{2}(k_+ + k_-) = \omega \left(1 - \frac{1}{2}\chi_0 \left(\frac{\omega_B}{\omega}\right)^2 - \frac{1}{8}\chi_0^2 \left(\frac{\omega_B}{\omega}\right)^2 + \dots \right) , \tag{9}$$

which engenders time delay. Unlike the familiar gyroelectric Faraday effect, in which electric charges are present, both the frequency dependence of the rotation and of the time delay are *trivial* in leading order in small quantities. At this order, the rotation angle, after transit through a length l, is

$$\phi_0 = \frac{g\mu_M}{2} \int_0^l \mathcal{M}_0(x) dx \,, \tag{10}$$

where $\mathcal{M}_0 = n_M \mu \mathcal{P}$ in a medium of spins of mass M, number density n_M, and polarization $\mathcal{P}$. We note that the rotation angle is a signed quantity and can tend to cancel if both particles and antiparticles are present.

Terrestrial studies are tenable in this case because (i) we can apply a strong magnetic field of known strength, (ii) Faraday rotation accrues coherently under momentum reversal, (iii) measurements of very small rotation angles are possible [28], and finally (iv) entry into the magnetic field itself acts as a longitudinal Stern-Gerlach device [29]. This last implies that we need not rely on any primordial polarization to detect an effect; rather, entry into a magnetic field region itself acts as a spin filter device. This technique is used to polarize ultra-cold neutrons (UCNs) with near 100% efficiency in the UCNA experiment at Los Alamos [30]. The "wrong" (higher energy) spin state cannot enter the magnetic field region if it has a sufficiently low kinetic energy. Since the magnetization is determined by energy considerations, it is unaltered upon the replacement of particle with antiparticle and thus by $\mu \to -\mu$ under the CPT theorem. However, under this replacement, the RHS of Eq. (10) changes sign. Faraday rotation probes the properties of the medium; it is not a single-particle probe. If the particle-antiparticle symmetry were perfect, the rotation angle would vanish. One can also establish a dark-matter magnetic moment through experiments which search for anomalous recoils from spin-dependent scattering, though these studies are insensitive to the sign of the magnetic moment. Presuming sensitivity to comparable magnetic moments and masses, such studies and Faraday rotation studies are complementary. In contradistinction to scattering experiments, the Faraday effect can be used to discover whether an *asymmetry* in the dark sector is indeed present.

4. Summary

The preponderance of matter is unknown, and we can probe its nature via its interactions with light. The discovery of dispersive effects in the speed of light in propagation from distant GRBs at large redshifts would signal the presence of dark matter. The discovery of a non-zero millicharge would demonstrate that dark matter is stable by dint of an internal $U(1)$ symmetry. Studies of dispersive effects at optical energies and beyond can constrain "wimpless" models and more.

We have also considered the possibility of observing a dark-matter candidate particle with a non-zero magnetic moment through the gyromagnetic Faraday effect. A non-zero Faraday rotation angle would signal that dark matter possesses a particle-antiparticle asymmetry — a unique insight.

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